Madras College Maths Department

Higher Maths

Logarithms and Exponentials

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Logarithms and Exponentials

A logarithm is the inverse function of an exponential. It can be used to solve the following equations.

Q) Solve the following:

a) $10^x = 100$

b) $10^x = 50$

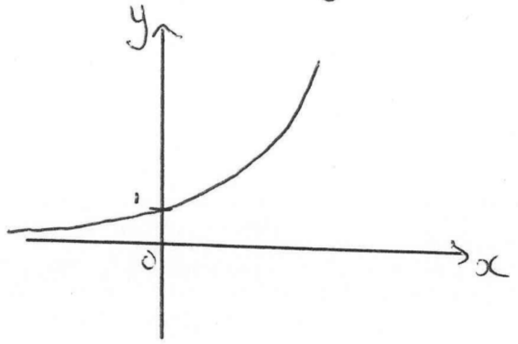
c) $7^x = 300$

Maths 3 Outcome 3

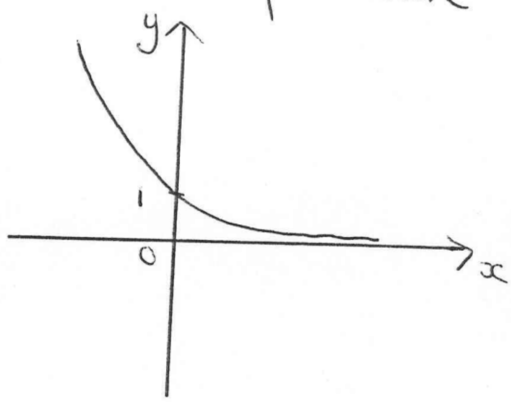
Exponential Growth and Decay

$y = a^x$ is called an exponential function to the base a. It is often used when modelling populations.

If $a > 1$ then
$y = a^x$ is a growth function



If $0 < a < 1$ $y = a^x$
is a decay function



Ex

Cells in a petri dish multiply at a rate of 20% per day. Taking C_0 as the initial population:

a) Find a formula for C_n, the number of cells after n days.

b) how long will it take for the number of cells to at least double.

A special Exponential Function

As n gets bigger the value of $\left(1+\frac{1}{n}\right)^n$ approaches a limit and this value is denoted by the letter e.

$e = 2.718281828....$ is another well known mathematical constant.

$f(x) = e^x$ is called the exponential function base e, it has the special property that its derivative is the same as it $\Rightarrow f(x) = f'(x) = e^x$.

Ex

A van hire company calculates the depreciation in the value of its vans using the formula $V(t) = V_0 e^{-0.16t}$. A new van costs £18000, calculate its value after 5 years.

Logarithms

Remember that the inverse of $y = a^x$ is called the logarithmic function.

$$\boxed{y = a^x \iff \log_a y = x}$$

Ex 1

Write in logarithmic form

(a) $3^4 = 81$

(b) $\frac{1}{2} = 2^{-1}$

Ex 2

Simplify

(a) $\log_2 4$

(b) $\log_4 64$

(c) $\log_3 \frac{1}{27}$

Laws of Logarithms

Law 1 $$\log_a xy = \log_a x + \log_a y$$

Proof

let $\log_a x = p$ and $\log_a y = q$

$\Rightarrow$ $x = a^p$ $y = a^q$

$$xy = a^p a^q$$

$$xy = a^{p+q}$$

put into log ~~exp~~ form

$$\log_a xy = p + q$$

sub back,

$$\log_a xy = \log_a x + \log_a y$$

Law 2 $$\log_a \frac{x}{y} = \log_a x - \log_a y$$

Proof

let $\log_a x = p$ and $\log_a y = q$

$\Rightarrow$ $x = a^p$ $y = a^q$

$$\frac{x}{y} = \frac{a^p}{a^q}$$

$$\frac{x}{y} = a^{p-q}$$

log form $$\log_a \frac{x}{y} = p - q$$

sub back $$\log_a \frac{x}{y} = \log_a x - \log_a y$$

Law 3 $\quad \log_a x^n = n\log_a x$

Proof let $\log_a x = p$

$\Rightarrow \quad x = a^p$

hence $\quad x^n = (a^p)^n$

$x^n = a^{np}$

log form $\quad \log_a x^n = np$

sub back $\quad \log_a x^n = n\log_a x$

It is also important to remember

$\log_a a = 1 \qquad (a^1 = a)$

$\log_a 1 = 0 \qquad (a^0 = 1)$

We use the laws of logs to manipulate algebraic expressions in order to simplify or solve them.

Ex 1 Simplify

(a) $\log_8 2 + \log_8 4$ (b) $3\log_3 3 + \frac{1}{2}\log_3 9$

Ex2

If $\log_a y = \log_a 2 + 3\log_a x$, express y in terms of x.

Natural Logarithms

Logarithms to the Base e are called natural logarithms, written $\log_e x$ or $\ln x$.

Ex 1

Solve these equations correct to 2 d.p.

(a) $4^x = 25$ (b) $e^{3x} = 35$ (c) $\ln x = 12$

Logarithmic Equations

Ex
Solve the following equations for $x > 0$

(a) $\log_a 4 + \log_a x = \log_a 12$

(b) $\log_a (x+1) + \log_a (x-1) = \log_a 8$

The number of pairs of breeding gulls in a nature reserve is given by $P(t) = 500(1.09)^t$, where t is the time in years since records began.

(a) How many pairs were there initially?

(b) After how many years will the population exceed 2000 pairs for the first time?

A number N_o of radioactive nuclei decay to N_t after t years according to the law $N_t = N_0 e^{-0.05t}$.

(a) Find the number remaining after 50 years if the original number N_o was 500.

(b) The half-life of a radioactive sample is defined as the time taken for the activity to be reduced by half. Calculate the half-life for this sample.

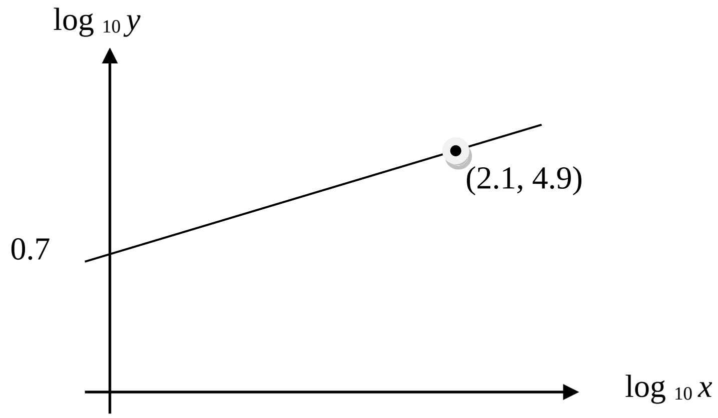
Interpreting Experimental Data

2 sets of data are often linked by exponential growth or decay. Logarithms can be applied to determine the equation of the function.

These functions can be of the form:

- $y = kx^n$
- $y = ab^x$

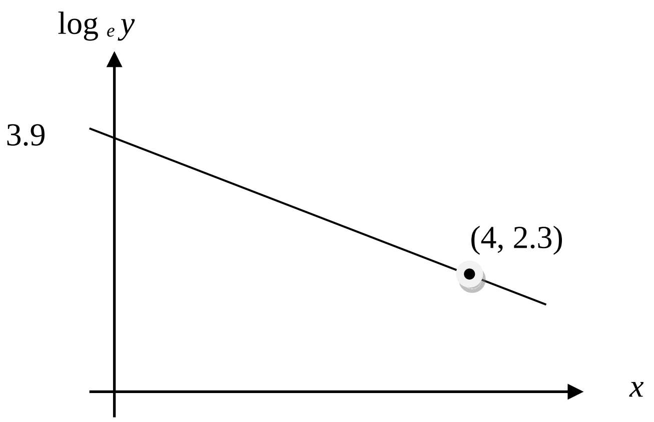
1) Results from an experiment are shown in the graph.



(a) Show this graph represents a relationship of the form $y = kx^n$

(b) Determine the values of k and n.

2) Results from an experiment are shown in the graph.



(a) Show this graph represents a relationship of the form $y = ab^x$

(b) Determine the values of a and b.